Problem 1 – Perturbation Expansion

For the system of oscillators given in Problem 1 of Problem Set 3:

(a) Show that it is possible to represent the terms in the perturbation expansion of the one particle Green’s function in powers of \( g \) by Feynman’s diagrams and state their structure.

(b) Prove that only one diagram contributes to the self-energy \( \Sigma(\omega_n) \) of the special oscillator.

(c) Derive by using Dyson’s equation the one particle Green’s function \( G(\omega_n) \).

Problem 2 – Interaction of fermions with a magnetic impurity

Fermions interacting with a single magnetic impurity are described by the Hamiltonian

\[
H = \int d^3 \vec{r} \psi^\dagger(\vec{r}) \left( -\frac{1}{2} \nabla^2 - \mu \right) \psi(\vec{r}) + \frac{1}{2} J \psi^\dagger(0) \vec{S} \sigma \psi(0)
\]

Here \( \vec{S} \) is the impurity spin, \( \sigma \) are the Pauli matrices and the definition of the Green’s function includes an average over the states of \( \vec{S} \).

(a) Write down the perturbation series for the one particle Green’s function of the given fermions up to the second order.

(b) Calculate the self-energy up to the second order.

Hint: An average of an observable \( A \) over the states of \( \vec{S} \) is given by

\[
\langle A \rangle = \frac{1}{2S+1} \sum_{m=-S}^{S} \langle S_m | A | S_m \rangle
\]
Problem 3

The one particle Green’s function for a weakly disordered system of fermions has the form

\[ G^R(k, E) = \frac{1}{E - \varepsilon_k + i/\tau} \]

Similar to Problem 1 of Problem set 1 find the real-space representation \( G^R(\vec{r}, E) \) in the limit \( \frac{1}{\tau} \ll E_F \), where \( E_F \) is the Fermi energy.

Hint: Consider that \( E \to E_F \)